

## **School of Mathematics, DAVV, Indore**

Taking into consideration the recent advances in different areas of mathematics, the board of studies in Mathematics, after discussions with experts, has revised the syllabus of M.Sc./M. A. Mathematics course.

### **Aims:**

1. Strengthening the logical reasoning which is the main ingredient to understand mathematical concepts.
2. Create more interest in the subject and motivate students for self learning.
3. Developing the mathematical skills among the students and preparing them to take up a career in research.

### **Objectives:**

1. To make students understand the techniques of proof in Mathematics and apply suitable techniques to tackle problems.
2. To inculcate the habit of making observations and experimentation and arrive at the final result.
3. Make student acquire the communication skill to present technical Mathematics so as to take up a career in Teaching Mathematics at various levels including schools, colleges, universities, etc.

### **General e-references:**

1. National Programme on Technology Enhanced Learning.(Mathematics)  
<http://www.nptelvideos.com/mathematics/>
2. Mathematics Video Lectures  
<http://freevidelectures.com/Subject/Mathematics>
3. MIT Open Course Ware  
<http://ocw.mit.edu/courses/audio-video-courses/>

e-references pertinent to each course are given at the end of the syllabus of the course.

**SCHEME OF EXAMINATION**  
**M.A. /M.Sc. Mathematics.**

**Semester I**

CODE	SUBJECT	L	T	C
M 111	Field theory	4	--	4
M 112	Real Analysis-I	4	--	4
M 113	Topology-I	4	--	4
M 114	Complex Analysis-I	4	--	4
M 101	Differential Equations-I	4	--	4
	Viva-Voce			4

Contact hours	:	20 per week
Valid credits	:	20
L	:	Lecture
T	:	Tutorial
C	:	Credits

\* The following notation comprising of three digits is used while numbering the courses.

1. The first digit refers to the semester number i.e. 1,2,3, and 4.
2. The second digit 1 refers to a core course and the second digit 0 refers to an optional course.
3. The third digit refers to the serial number of the course.

# **SCHEME OF EXAMINATION**

## **M.A. /M.Sc. Mathematics**

### **Semester II**

<b>CODE</b>	<b>SUBJECT</b>	<b>L</b>	<b>T</b>	<b>C</b>
M 211	Advanced Abstract Algebra	4	--	4
M 212	Real Analysis-II	4	--	4
M 213	Topology-II	4	--	4
M 214	Complex Analysis-II	4	--	4
M 201	Differential Equations-II	4	--	4

Viva-Voce 4

Contact hours	:	20 per week
Valid credits	:	20
L	:	Lecture
T	:	Tutorial
C	:	Credits

\* The following notation comprising of three digits is used while numbering the courses.

1. The first digit refers to the semester number i.e. 1,2,3, and 4.
2. The second digit 1 refers to a core course and the second digit 0 refers to an optional course.
3. The third digit refers to the serial number of the course.

# SCHEME OF EXAMINATION

## M.A. /M.Sc. Mathematics

### Semester III

CODE	SUBJECT	L	T	C
M 311	Integration Theory	4	-	4
M 312	Functional Analysis	4	-	4
M 313	Partial Differential Equations	4	-	4
M 301	Theory of Linear Operators-I	4	-	4
M 302	Linear Programming-I	4	-	4
M305	Mathematical Modelling-I	3		3
	Viva-Voce			4

Contact hours	:	23 per week
Valid credits	:	23
L	:	Lecture
T	:	Tutorial
C	:	Credits

\* The following notation comprising of three digits is used while numbering the courses.

1. The first digit refers to the semester number i.e. 1,2,3, and 4.
2. The second digit 1 refers to a core course and the second digit 0 refers to an optional course.
3. The third digit refers to the serial number of the course.
4. M305 is an elective generic course.

# SCHEME OF EXAMINATION

## M.A. /M.Sc. Mathematics

### Semester IV

CODE	SUBJECT	L	T	C
M 411	Mechanics	4	-	4
M 401	Theory of Linear Operators - II	4	-	4
M 402	Linear Programming -II	4	-	4
M 403	Homotopy Theory	4	-	4
M 404	Topics In Ring Theory	4	-	4
M405	Mathematical Modelling-II	3	--	3
	Viva-Voce			4

Contact hours	:	23 per week
Credits	:	23
L	:	Lecture
T	:	Tutorial
C	:	Credits

\* The following notation comprising of three digits is used while numbering the courses.

1. The first digit refers to the semester number i.e. 1,2,3, and 4.
2. The second digit 1 refers to a core course and the second digit 0 refers to an optional course.
3. The third digit refers to the serial number of the course.
4. M405 is an elective generic course.

# CORE COURSES

## Semester I

- M 111 Field theory
- M 112 Real Analysis-I
- M 113 Topology-I
- M 114 Complex Analysis-I

## Semester II

- M 211 Advanced Abstract Algebra
- M 212 Real Analysis-II
- M 213 Topology-II
- M 214 Complex Analysis-II

## Semester III

- M 311 Integration Theory
- M 312 Functional Analysis
- M 313 Partial Differential Equations

## Semester IV

- M 411 Mechanics

# **ELECTIVE COURSES(Discipline Centric)**

## **Semester I**

<b>CODE</b>	<b>SUBJECT</b>
M 101	Differential Equations-I
M 102	Advanced Discrete Mathematics-I
M 103	Differential Geometry of Manifolds-I

## **Semester II**

<b>CODE</b>	<b>SUBJECT</b>
M 201	Differential Equations-II
M 202	Advanced Discrete Mathematics-II
M 203	Differential Geometry of Manifolds-II

## **Semester III**

<b>CODE</b>	<b>SUBJECT</b>
M 301	Theory of Linear Operators-I
M 302	Linear Programming-I
M 303	Programming in C – Theory & Practical
M 304	Mathematics of Finance & Insurance-I

## **Semester IV**

<b>CODE</b>	<b>SUBJECT</b>
M 401	Theory of Linear Operators - II
M 402	Linear Programming-II
M 403	Homotopy Theory
M 404	Topics In Ring Theory
M 405	Algebraic Topology
M 406	Analytical Number Theory
M 407	Abstract Harmonic Analysis
M 408	Mathematics of Finance & Insurance-II

\* Those electives will be offered for which expertise is available in the department.

## **ELECTIVE COURSE(Generic)**

M305 Mathematical Modelling-I  
M405 Mathematicsl Modelling-II



# SYLLABUS

## M.A./M.Sc. MATHEMATICS

### SEMESTER I

- M 111 Field theory
- M 112 Real Analysis-I
- M 113 Topology-I
- M 114 Complex Analysis-I
- M 101 Differential Equations-I

### SEMESTER II

- M 211 Module Theory
- M 212 Real Analysis-II
- M 213 Topology-II
- M 214 Complex Analysis-II
- M 201 Differential Equations-II

### SEMESTER III

- M 311 Integration Theory
- M 312 Functional Analysis
- M 313 Partial Differential Equations
- M 301 Theory of Linear Operators-I
- M 302 Linear Programming-I

### SEMESTER IV

- M 411 Mechanics
- M 401 Theory of Linear Operators - II
- M 402 Linear Programming -II
- M 403 Homotopy Theory
- M 404 Topics in Ring Theory

**Course Plan:** Each course has five units and each unit shall be covered in three weeks on average.

## SEMESTER - I

### M 111 FIELD THEORY

Pre-requisites: Basics on rings and fields.(relevant parts of reference [1])

#### **Unit I:**

Finite & Algebraic extensions, Algebraic closure,

#### **Unit II:**

splitting fields and normal extensions, separable extensions,

#### **Unit III:**

Finite fields, Primitive elements and purely inseparable extensions.

#### **Unit IV:**

Galois extensions, examples and applications,

#### **Unit V:**

Roots of unity, Linear independence of characters, cyclic extensions, solvable & radical extensions.

#### **Book Recommended:**

1. Serge Lang : Algebra

#### **Reference Books:**

1. I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
2. M.Artin, Algebra, Prentice - Hall of India, 1991.
3. N.Jacobson, Basic Algebra, vols I&II, W.H.Freeman,1980.
4. S.Kumaresan, Linear Algebra, A Geometric Approach, Prentice - Hall of India, 2000.
5. D.S.Dummit & R.M.Foote, Abstract Algebra, II Ed, John Wiley & Sons, Inc, New York.

#### **E-references:**

1. Notes on Galois Theory, Sudhir R. Ghorpade  
Department of Mathematics, Indian Institute of Technology, Bombay 400 076  
<http://www.math.iitb.ac.in/~srg/Lecnotes/galois.pdf>
2. Galois Theory, Dr P.M.H. Wilson.  
<http://www.jchl.co.uk/maths/Galois.pdf>

**M 112**  
**REAL ANALYSIS-I**

**Pre-requisites:** Chapters 1 to 5 of reference [1]

**Unit I:**

The Riemann-Stieltjes Integral Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector-valued functions, rectifiable curves.

**Unit II:**

Sequences and Series of Functions Rearrangements of terms of a series, Riemann theorem, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, uniform convergence and continuity, uniform convergence and integration.

**Unit III:**

Uniform convergence and differentiation, equicontinuous families of functions, the Stone-Weierstrass Theorem, uniform convergence and Riemann-Stieltjes integral,

**Unit IV:**

Abels test for uniform convergence, Dirichlet's test for uniform convergence. power series, Abel's theorem.

**Unit V:**

Functions of Several Variables, derivatives in an open subset of  $\mathbb{R}^n$ , chain rule, partial derivatives, interchange of the order of differentiation, derivatives of higher orders, inverse function theorem, implicit function theorem, Jacobians, differentiation of integrals, Taylor's theorem, extremum problems with constraints, Lagrange's multiplier method.

**Books Recommended:**

1. Walter Rudin, Principles of Mathematical Analysis(3rd edition), McGraw-Hill international editions, 1976.
2. T.M. Apostol, Mathematical Analysis(2nd edition), Narosa Publishing House, New Delhi, 1989.

**Reference Books:**

1. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 1966.
2. H.L. Royden, Real Analysis(4th edition), Macmillan Publishing Company, 1993.

# M 113

## TOPOLOGY – I

### Unit I: (Pre-requisites)

Relations, Countable and uncountable sets. Infinite sets and the Axiom of choice. Cardinal numbers and their arithmetic. Schroeder-Bernstein theorem. Cantor's theorem and continuum hypothesis. Zorn's lemma. Well-ordering theorem.

### Unit II:

Topological spaces. The order topology. Product topology on  $X \times Y$ . Bases and subbases. Subspaces and relative topology.

### Unit III:

Closed sets and limit points. Closure of a set. Dense subsets. Interior, exterior and boundary of a subset.

### Unit IV:

Continuous Functions and homeomorphisms. The product topology. The metric topology. The quotient topology

### Unit V:

Connected spaces. Connectedness on the real line. Components. Locally connected spaces. Connectedness and product spaces

### Books Recommended :

1. James R. Munkres, Topology (Second edition), Prentice-hall of India
2. George F. Simmons, Introduction to topology and modern analysis, McGraw Hill Book Company Inc.

### References :

1. N. Bourbaki, General topology, Springer-verlag.
2. K.D.Joshi, Introduction to topology, Wiley Eastern.
3. J.L.Kelley, General topology, Affiliated East-West press Pvt Ltd.

### E-references:

1. Topology Course Lecture Notes Aisling McCluskey and Brian McMaster  
<http://at.yorku.ca/i/a/a/b/23.htm>
2. Notes on Introductory Point-Set Topology, Allen Hatcher  
<http://www.math.cornell.edu/~hatcher/Top/TopNotes.pdf>
3. Introduction to Topology, Renzo  
<http://www.math.colostate.edu/~renzo/teaching/Topology10/Notes.pdf>
4. Topology Lecture Notes, Thomas Ward, UEA  
<http://www.uea.ac.uk/~h720/teaching/topology/materials/topology.pdf>

**M 114**  
**COMPLEX ANALYSIS-I**

**Pre-requisites:**

Basic Metric space theory: relevant parts of reference [1]

**Unit I:** Algebra of complex numbers, geometric aspects like equations to straight lines, circles, analytic functions, exponential, trigonometric, hyperbolic functions, Branches of many valued functions with special reference to  $\arg z$ ,  $\log z$ , and complex exponents.

**Unit II:**

Complex integration, Cauchy's Theorem, Cauchy's Integral formula, Higher ordered derivatives, Morera's theorem, Liouville's theorem, Fundamental theorem of algebra.

**Unit III:**

Taylor's theorem, Maximum Modulus Principle, Schwarz Lemma.

**Unit IV:**

Isolated singularities, Meromorphic functions, Laurent's series, Argument Principle, Rouché's theorem.

**Unit V:**

Residues, Cauchy's Residue theorem, evaluation of integrals, their properties and classification, definitions and examples of conformal mappings, Hadamard three circles theorem.

**Books Recommended:**

1. J.B. Conway : Functions of one Complex variable, Springer-verlag.
2. S. Ponnusamy : Foundations of Complex Analysis, Narosa Pub, '97

**References :**

1. George F. Simmons, Introduction to topology and modern analysis, McGraw Hill Book Company Inc.
2. Alfohrs : Complex Analysis

**E-references:**

1. Introduction to Complex Analysis  
<http://rutherglen.science.mq.edu.au/wchen/lnicafolder/lnica.html>
2. Complex Analysis  
[www.umn.edu/~arnold/502.s97/complex.pdf](http://www.umn.edu/~arnold/502.s97/complex.pdf)

**M 101**  
**DIFFERENTIAL EQUATIONS-I**

**Pre-requisites:** Chapters 1 to 5 of reference [2]

**Unit I:**

Initial value problems and the equivalent integral equation,  $n$ th order equation in  $d$ -dimensions as a first order system, concepts of local existence, existence in the large and uniqueness of solutions with examples. Basic theorems: Ascoli- Arzela theorem, a theorem on convergence of solutions of a family of initial value problems. Picard-Lindelof theorem, Peano's existence theorem and corollary

**Unit II:**

Maximal interval of existence, Extension theorem and corollaries, Kamke's convergence theorem, Kneser's theorem. (Statement only) Differential inequalities and uniqueness : Gronwall's inequality, Maximal and minimal solutions, Differential inequalities.

**Unit III:**

A theorem of Wintner, Uniqueness theorems, Nagumo's and Osgood's criteria. Egres points and Lyapunov functions, Successive approximations.

**Unit IV:**

Linear differential equations : Linear systems, Variation of constants, reduction to smaller systems, Basic inequalities, constant coefficients.

**Unit V:**

Floquet theory, adjoint systems, Higher order equations. Dependence on initial conditions and parameters: Preliminaries, continuity, and differentiability.

**Books Recommended :**

1. P.Hartman, Ordinary differential equations, John Wiley, 1964.
2. Walter Rudin, Principles of Mathematical Analysis, (3rd edition), McGraw-Hill international editions, 1976.

**References :**

1. W.T.Reid, Ordinary differential equations, John Wiley & sons, New York, 1971.
2. E.A.Coddigton & N.Levinson, Theory of ordinary differential equations, McGraw Hill, NY, 1955.

**E-references:**

1. Ordinary Differential Equations, Modern Perspective Mohan C Joshi IITBombay  
<http://www.math.iitb.ac.in/~mcj/root.pdf>
2. Differential Equations, Paul Dawkins  
[http://tutorial.math.lamar.edu/pdf/DE/DE\\_Complete.pdf](http://tutorial.math.lamar.edu/pdf/DE/DE_Complete.pdf)
3. Ordinary Differential Equations-Lecture Notes, Eugen J. Ionascu  
<http://math.columbusstate.edu/ejionascu/papers/diffeqbook.pdf>

## SEMESTER – II

### M 211 MODULE THEORY

**Pre-requisites:** Basics on rings and fields.(relevant parts of reference [1])

**Unit I:**

Modules: Basic definitions, direct products and sums, Free modules.

**Unit II:**

Noetherian rings and modules, Hilbert Basis Theorem, Power series, Associated primes, primary decomposition.

**Unit III:**

Modules over PID's, decomposition over one endomorphism, Characteristic polynomial, Jordan & Rational canonical forms.

**Unit IV:**

Semisimplicity: Matrices & Linear maps over non-commutative rings, conditions defining semisimplicity, the Density theorem,

**Unit V:**

semisimple rings & simple rings .Representations of finite groups.

**Book Recommended:**

1. Serge Lang : Algebra

**Reference Books:**

1. I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

2. M.Artin, Algebra, Prentice - Hall of India, 1991.

3. N.Jacobson, Basic Algebra, vols I&II, W.H.Freeman,1980.

4. S.Kumaresan, Linear Algebra, A Geometric Approach, Prentice - Hall of India

5. D.S.Dummit & R.M.Foote, Abstract Algebra, II Ed, John Wiley & Sons, Inc, New York.

**REAL ANALYSIS-II**

**Pre-requisites:** Algebra of sets,  $\sigma$  algebra, Riemann integration

**Unit I:**

Lebesgue outer measure, measurable sets, a non measurable set

**Unit II:**

Hausdorff measures on the real line, Hausdorff dimension, Hausdorff dimensions of a Cantor like set.

**Unit III:**

measurable functions, Littlewoods three principles, a non Borel measurable set, Egoroff's theorem

**Unit IV:**

the Lebesgue integral of a bounded function over a set of finite measure, the integral of a nonnegative function, the general Lebesgue integral, properties of these integrals, convergence theorems.

**Unit V:**

Differentiation of monotone functions, functions of bounded variation, differentiation of an integral, absolute continuity, convex functions, the  $L_p$  spaces, the Minkowski and Holder inequalities, convergence and completeness, Bounded linear functionals on  $L_p$  spaces..

**Books Recommended:**

1. H.L. Royden, Real Analysis(4th edition), Macmillan Publishing Company, 1993.

**References:**

1. G. de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.

2. Lebesgue Measure and integration: an introduction, Frank Burk, Wiley Interscience Publication, 1998.

3. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.



## E-references:

1. <http://www.math.dauniv.ac.in/Dr.MaheshNDumaldar.php>
2. Review of Lebesgue Measure and Integration, Christopher E. Heil, School of Mathematics Georgia Institute of Technology  
<http://people.math.gatech.edu/~heil/handouts/real.pdf>
3. Measure Theory and Lebesgue Integration, Joshua H. Lifton  
[http://web.media.mit.edu/~lifton/snippets/measure\\_theory.pdf](http://web.media.mit.edu/~lifton/snippets/measure_theory.pdf)
4. The Lebesgue Measure and Integral, Mike Klaas  
<http://www.cs.ubc.ca/~klaas/research/lebesgue.pdf>

**M 213**  
**TOPOLOGY – II**

**Pre-requisites: M113**

**Unit I:**

Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and Countably compact sets.

**Unit II:**

Local compactness and one-point compactification. Compactness in metric spaces. Equivalence of compactness, Countable compactness and Sequential compactness in metric spaces. Nets.

**Unit III:**

First and second countable spaces. Lindelof's theorems. Second countability and separability. Countability and product spaces. Separation axioms.  $T_0$ ,  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ ; their characterization and basic properties. Urysohn's lemma.

**Unit IV:**

Tietze extension theorem. Urysohn (metrization) embedding theorem. Separation axioms and product spaces. The Tychonoff's theorem. Stone-Cech compactification. Metrization theorems and paracompactness: Local finiteness. The Nagata-Smirnov metrization theorem. Paracompactness. The Smirnov metrization theorem.

**Unit V:**

The fundamental group and covering spaces : Homotopy of paths. The fundamental group. Covering spaces. The fundamental group of the circle and the fundamental theorem of algebra.

**Books Recommended :**

1. James R. Munkres, Topology (Second edition), Prentice-hall of India
2. George F. Simmons, Introduction to topology and modern analysis, McGraw Hill Book Company Inc.

**References :**

1. N. Bourbaki, General topology, Springer-verlag.
2. K.D.Joshi, Introduction to topology, Wiley Eastern.
3. J.L.Kelley, General topology, Affiliated East-West press Pvt Ltd.

**E-references:**

1. Topology Course Lecture Notes Aisling McCluskey and Brian McMaster  
<http://at.yorku.ca/i/a/a/b/23.htm>
2. Notes on Introductory Point-Set Topology, Allen Hatcher  
<http://www.math.cornell.edu/~hatcher/Top/TopNotes.pdf>
3. Introduction to Topology, Renzo  
<http://www.math.colostate.edu/~renzo/teaching/Topology10/Notes.pdf>
4. Topology Lecture Notes, Thomas Ward, UEA  
<http://www.uea.ac.uk/~h720/teaching/topology/materials/topology.pdf>

**COMPLEX ANALYSIS-II**

**Pre-requisites:** Basic Metric space theory: relevant parts of reference [1]

**Unit I:**

Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weierstrass Factorisation theorem.

**Unit II:** Gamma function & its properties, Riemann Zeta function, Riemann's functional equation, Runge's theorem and Mittag-Leffler's theorem.

**Unit III:** Analytic continuation, uniqueness of direct analytic continuation and analytic continuation along a curve, power series method of analytic continuation, Schwartz Reflection Principle, Monodromy theorem and its consequences.

**Unit IV:** Harmonic functions on a disk, Dirichlet problem, Green's function.

**Unit V:** Canonical products, Jensen's formula, order of an entire function, exponent of convergence, Hadamard's factorization theorem, range of an analytic function, Bloch's theorem, The Little Picard theorem, Schottky's theorem, Great Picard theorem.

**Books Recommended:**

1. J.B. Conway : Functions of one Complex variable, Springer-verlag.
2. S. Ponnusamy : Foundations of Complex Analysis, Narosa Pub, '97

**References :**

1. Ahlfors : Complex Analysis

**E-references:**

1. Introduction to Complex Analysis  
<http://rutherglen.science.mq.edu.au/wchen/lnicafolder/lnica.html>
2. Complex Analysis  
[www.umn.edu/~arnold/502.s97/complex.pdf](http://www.umn.edu/~arnold/502.s97/complex.pdf)

## M 201

### DIFFERENTIAL EQUATIONS-II

**Pre-requisites: M101**

**Unit I:**

Poincare- Bendixson theory: Autonomous systems, Umlanfsatz, index of a stationary point, Poincare- Bendixson theorem, stability of periodic solutions,

**Unit II:**

rotation points, foci, nodes and saddle points. Linear second order equations : Preliminaries, Basic facts, Theorems of Sturm,

**Unit III:**

Sturm-Liouville boundary value problems, Number of zeros,

**Unit IV:**

Nonoscillatory equations and principal solutions, Nonoscillation theorems.

**Unit V:**

Use of Implicit function and fixed point theorems : Periodic solutions, linear equations, nonlinear problems.

**Books recommended :**

1. P.Hartman, Ordinary differential equations, John Wiley, 1964.

**References :**

1. W.T.Reid, Ordinary differential equations, John Wiley & sons, New York,1971.
2. E.A.Coddigton & N.Levinson, Theory of ordinary differential equations, McGraw Hill, NY, 1955.

**E-references :**

1. Ordinary Differential Equations, Modern Perspective Mohan C Joshi IITBombay  
<http://www.math.iitb.ac.in/~mcj/root.pdf>
2. Differential Equations, Paul Dawkins  
[http://tutorial.math.lamar.edu/pdf/DE/DE\\_Complete.pdf](http://tutorial.math.lamar.edu/pdf/DE/DE_Complete.pdf)
3. Ordinary Differential Equations-Lecture Notes, Eugen J. Ionascu  
<http://math.columbusstate.edu/ejonascu/papers/diffeqbook.pdf>

## SEMESTER – III

### M 311

## INTEGRATION THEORY

**Pre-requisites:** Lebesgue Measure theory [1]

### Unit I:

Measure spaces, Measurable functions, Integration, Convergence theorems.

**Unit II:** Signed measures, The Radon-Nikodym theorem, Lebesgue decomposition,  $L^p$  spaces, Riesz representation theorem.

[1] Chapter 11

**Unit III:** Outer measure and measurability, The extension theorem, Lebesgue- Steiltjes integral, Product measures, Fubini's theorem.

[1] Chapter 12, Sections 1,2,3,4.

**Unit IV:** Baire sets, Baire Measure, Continuous functions with compact support, Regularity of measures on locally compact spaces.

**Unit V:** Integration of continuous functions with compact support, Riesz- Markoff theorem.

[1] Chapter 13.

### Recommended Books :

1. H.L. Royden, Real Analysis, Mc millan Pub. Co. Inc. New York, 4<sup>th</sup> Edition, 1993.
2. G.de.Barra., Measure Theory and Integration, Wiley Eastern Limited, 1981
3. Inder K. Rana. An introduction to Measure & Integration Narosa Pub. House, Delhi, 1997.
4. P.K. Jain, N.P. Gupta, Lebesgue Measure and Interation New Age International (P) Ltd., New Delhi, 1986.

### E-references:

1. Notes on measure and integration in locally compact – Mathematics

<http://math.berkeley.edu/~arveson/Dvi/rieszMarkov.pdf>

**FUNCTIONAL ANALYSIS**

**Pre-requisites:** Metric spaces, compactness, connectedness.

**Unit I:**

Completion of a metric space, Normed linear spaces. Banach spaces and examples. Quotient space of normed linear space and its completeness.

**Unit II:**

Equivalent norms. Riesz lemma, basic properties of finite dimensional normed linear space and compactness. Weak convergence and bounded.

**Unit III:**

linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples. Open mapping and closed graph theorems. Uniform boundedness theorem and some of its consequences, the Hahn-Banach theorem.

**Unit IV:**

Reflexive spaces. Weak sequential compactness.

**Unit V:**

Inner product spaces. Hilbert spaces. Orthonormal sets. Bessel's inequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem, Riesz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces. Self-adjoint operators, positive, projection, normal and unitary operators.

**Recommended Books :**

1. H.L. Royden, Real Analysis, Macmillan Publishing Co. Inc. New York, 4th Edition, 1993.
2. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw- Hill Book Company, New York, 1963.
3. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons. New York, 1978
4. B.V. Limaye, Functional Analysis, New Age Intergration (P) Ltd., 1996

**E-references:**

1. An introduction to some aspects of functional analysis, Stephen Semmes, Rice University <http://math.rice.edu/~semmes/fun2.pdf>
2. <http://www.math.dauniv.ac.in/Dr.MaheshNDumaldar.php>



**PARTIAL DIFFERENTIAL EQUATIONS****Unit I:**

Examples of PDE. Classification. Transport Equation-Initial Value Problem. Non-homogeneous Equation.

**Unit II:**

Laplace's Equation - Fundamental Solution, Mean Value Formulas, Properties of Harmonic Functions, Green's Function, Energy Methods. Heat Equation- Fundamental Solution, Mean Value Formula, Properties of Solutions, Energy Methods.

**Unit III:**

Wave Equation-Solution by Spherical Means, Non-homogeneous Equations, Energy Methods.

[1] Chapter 1 Art. 1.1, 1.2, Chapter. 2

**Unit IV:**

Nonlinear First Order PDE-Complete Integrals, Envelopes, Characteristics,

**Unit V:**

Hamilton-Jacobi Equations (Calculus of Variations, Hamilton's ODE, Legendre Transform, Hopf-Lax Formula, Weak Solutions, Uniqueness),

[1] Chapter. 3 Art. 3.1 to 3.3

**Recommended Books:**

1. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Volume 19, AMS, 1998.

**Reference Books:**

1. I. N. Sneddon, Elements of Partial Differential Equations, Mc Graw-Hill Book Company, 1985.
2. F. John, Partial Differential Equations, Third Edition, Narosa Publishing House, New Delhi, 1979.
3. P. Prasad and R. Ravidran, Partial Differential Equations, New Age International (P) Limited, Publishers, 1985.

**E-references:**

1. PARTIAL DIFFERENTIAL EQUATIONS MA 3132 LECTURE NOTES, B. Neta  
(<http://www.math.nps.navy.mil/~bneta/pde.pdf>)
2. Notes for Partial Differential Equations, Kuttler  
(<http://www.math.byu.edu/~klkuttler/547notesB.pdf>)



**THEORY OF LINEAR OPERATORS-I**

**Pre-requisites:** Basic linear algebra and basic functional analysis.

**Unit I:**

Spectral theory of normed linear spaces, resolvent set and spectrum, spectral properties of bounded linear operators, properties of resolvent and spectrum.

[1] Chapter 7

**Unit II:**

spectral mapping theorem for polynomials, spectral radius of bounded linear operator on a complex Banach space, elementary theory of Banach Algebras. [1] Chapter 7

**Unit III:**

Basic properties of compact linear operators. [1] Chapter 8

**Unit IV:**

Behaviour of Compact linear operators with respect to solvability of operator equations, Fredholm type theorems, Fredholm alternative theorem, Fredholm alternative for integral equations. [1] Chapter 8

**Unit V:**

Spectral properties of bounded self-adjoint linear operators on a complex Hilbert space, positive operators, Monotone sequence theorem for bounded self-adjoint operators on a complex Hilbert space, square roots of a positive operator, projection operators.

[1] Chapter 9, Sec 9.1-9.6

**Recommended Book:**

1. E.Kreyszig, Introductory Functional Analysis with applications, John-Wiley & Sons, New York, 1978.

**Reference Books:**

2. N.Dunford & J.T.Schwartz, Linear operators-3 parts, Interscience Wiley, New York, 1958-71.

3. P.R. Halmos, Introduction to Hilbert spaces and the theory of spectral multiplicity, second edition, Chelsea Pub. Co., N.Y. 1957.

4. P. R. Halmos, A Hilbert space problem book, D. Von Nostrand company Inc., 1967.

**Linear Programming - I**

**Pre-requisites:** Finite dimensional vector spaces, Linear transformations, Linear system of equations, basic solutions

**Unit I:**

Inverse of matrix by partition, product form of the inverse, basis to basis lemma, lines & hyperplanes, Convex sets and hyperplanes, polyhedral sets, extreme points, faces, directions and extreme directions, Decomposition theorem for polyhedra, Farkas' lemma.

**Unit II:**

Theory of simplex method, reduction of any feasible solution to a basic feasible solution, improving a basic feasible solution, unbounded solutions, optimality conditions, alternate optima, extreme points and basic feasible solutions,

**Unit III:**

computational aspects of the simplex method, initial basic feasible solution, inconsistency and redundancy, review of the simplex method, Big M method, two phase method

**Unit IV:**

resolution of degeneracy, Charne's perturbation method, Blande's rule, Revised simplex method, the simplex method for bounded variables.

**Unit V:**

Network Analysis: Shortest path problem, minimum spanning tree problem, Maximum flow problem, Minimum cost flow problem, Network simplex method.

**Recommended Books:**

1. G. Hadley, Linear programming, Narosa Publishing House, 1995.
2. Mokhtar S. Bazaraa, John J. Jarvis, Hanis D. Sherali, Linear Programming And Network Flows, 2nd Ed, Wiley-India, 2008
3. Schrijver, A.: Theory of Linear and Integer Programming, J. Wiley, 1986

**Reference Books:**

1. Dantzig G., Thapa M. Linear programming. Vol.1.. Introduction, Springer,1997
2. G.B. Dantzig, M. Thapa: Linear Programming 2: Theory and Extensions, Springer Verlag, 2003
3. G.Hadley, Linear Algebra, Addison – Wesley Pub. Co. Read. Mass. 1961
4. Nemhauser G.L., Wolsey L.A., Integer and combinatorial optimization, Wiley, NewYork, 1988
5. Chvatal V., Linear programming, Freeman, NewYork, 1983
6. Vanderbei R.J., Linear programming: Foundations and extensions, Kluwer Academic Publishers, Boston, MA, 1996

**E-references:**

1. <http://www.math.dauniv.ac.in/Dr.MaheshNDumaldar.php>
2. [Activity Analysis of Production and Allocation, Proceedings of a Conference, TJALLING C. KOOPMANS](#)
3. [National Programme on Technology Enhanced Learning.\(Mathematics\)](#)

## SEMESTER – IV

### M 411

### MECHANICS

#### Unit I:

Generalized coordinates. Holonomic and Non-holonomic Systems. Scleronomic and Rheonomic Systems. Generalized potential.

#### Unit II:

Lagrange's equations of first kind. Lagrange's equations of second kind. Uniqueness of solution. Energy equation for conservative fields. Hamilton's variables. Donkin's theorem. Hamilton canonical equations. Cyclic coordinates. Routh's equations.

#### Unit III:

Motivating problems of calculus of variations, Shortest distance. Minimum surface of revolution. Brachistochrone problem. Isoperimetric problem. Geodesic. Fundamental lemma of calculus of variations. Euler's equation for one dependent function and its generalization to (i) 'n' dependent functions (ii) higher order derivatives. Conditional extremum under geometric constraints and under integral constraints.

#### Unit IV:

Hamilton's principle, Principle of least action.

[2] Art. 1-6 & relevant parts of 8, 11-14, 16,20, [3] Relevant portion for Calculus of variations.

#### Unit V:

Poincare Cartan integral invariant. Whittaker's equations. Jacobi's equations. Statement of Lee Hwa Chung's theorem. Hamilton-Jacobi equation. Jacobi theorem. Method of separation of variables. Lagrange brackets. Poisson's bracket. Poisson's identity. Jacobi-Poisson Theorem. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

[2] Art. 15, 18, 20, 22, 24-27, 30, 32.

### **Books Recommended :**

1. F. Gantmacher, Lectures in Analytic Mechanics, MIR Publishers, Moscow, 1975.

### **Reference Books:**

1. H. Goldstein, Classical Mechanics (2<sup>nd</sup> Edition), Narosa Publishing House, New Delhi.
2. Narayan Chandra Rana and Pramod Sharad Chandra Joag, Classical Mechanics, Tata McGraw Hill, 1991.
3. Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University Press, 1998.

**THEORY OF LINEAR OPERATORS – II**

**Pre-requisites:** M 301

**Unit I:**

Spectral family of a bounded self-adjoint linear operator and its properties, spectral representation of bounded self-adjoint linear operators, spectral theorem

[1] Chapter 9, Sec 9.7-9.11

**Unit II:**

Unbounded linear operators in Hilbert space, Hellinger-Toeplitz theorem. [1] Chapter 10

**Unit III:**

Hilbert adjoint operators, Symmetric and self-adjoint linear operators, closed linear operators and closures, spectrum of an unbounded self-adjoint linear operator.

[1] Chapter 10

**Unit IV:**

spectral theorem for unitary and self adjoint linear operators. [1] Chapter 10

**Unit V:**

Multiplication operator and Differentiation operator. [1] Chapter 10

**Recommended Book:**

1 E.Kreyszig, Introductory Functional Analysis with applications, John-Wiley & Sons, New York, 1978.

**Reference Book:**

2 N.Dunford & J.T.Schwartz, Linear operators-3 parts, Interscience Wiley, New York, 1958-71.

## M 402

### Linear Programming –II

**Pre-requisites:** M401

#### **Unit I:**

Duality theory, weak and strong duality theorems, Complementary slackness, dual simplex method, primal dual algorithm, Integer programming, the KKT conditions.

#### **Unit II:**

Transportation problems, properties of the activity matrix of a transportation problem, simplification of the simplex method to the transportation problem, bases in a transportation problem, the stepping stone algorithm, resolution of degeneracy, determination of an initial basic feasible solution, alternative procedure for computing  $z_{ij} - c_{ij}$  using duality (u-v method).

#### **Unit III:**

Assignment problems, reduced cost coefficient matrix, the Hungarian method, Konig – Evergary theorem, The Birkoff –von Neumann theorem.

#### **Unit IV:**

Sensitivity analysis, the Dantzig-Wolf decomposition, Game theory and linear programming, Two person zero sum games, Games with mixed strategies, Graphical solution, Solution by linear programming.

#### **Unit V:**

Applications: Optimal product mix and activity levels, Petroleum refinery operations, Blending problems, Economic interpretation of dual linear programming problems, Input output analysis, Leontief systems.

**Recommended Books:**

1. G. Hadley, Linear programming, Narosa Publishing House, 1995.
2. Mokhtar S. Bazaraa, John J. Jarvis, Hanis D. Sherali, Linear Programming And Network Flows, 2Nd Ed, Wiley-India, 2008
3. Schrijver,A.: Theory of Linear and Integer Programming, J. Wiley, 1986

**Reference Books:**

1. Dantzig G., Thapa M. Linear programming. Vol.1.. Introduction, Springer,1997
2. G.B. Dantzig, M. Thapa: Linear Programming 2: Theory and Extensions, Springer Verlag, 2003
3. G.Hadley, Linear Algebra, Addison – Wesley Pub. Co. Read. Mass. 1961
4. Nemhauser G.L., Wolsey L.A., Integer and combinatorial optimization, Wiley, NewYork, 1988
5. Chvatal V., Linear programming, Freeman, NewYork, 1983
6. Vanderbei R.J., Linear programming: Foundations and extensions, Kluwer Academic Publishers, Boston, MA, 1996

**E-references:**

1. <http://www.math.dauniv.ac.in/Dr.MaheshNDumaldar.php>
2. [Activity Analysis of Production and Allocation, Proceedings of a Conference, TJALLING C. KOOPMANS](#)
3. [National Programme on Technology Enhanced Learning.\(Mathematics\)](#)



**HOMOTOPY THEORY**

**Pre-requisites: M113 & M213**

**Unit I:**

The Fundamental Group, Homotopic Paths and the Fundamental Group.

**Unit II:**

The Covering Homotopy Property for  $S^1$ , Examples of Fundamental Groups.

[1] Chapter 4.

**Unit III:**

Covering Spaces, The Definition and Some Examples, Basic Properties of Covering Spaces, Classification of Covering Spaces, Universal Covering Spaces, Applications. [1] Chapter 5.

**Unit IV:**

The Higher Homotopy Groups, Equivalent Definitions of  $\pi_n(X, x_0)$ , Basic Properties and Examples.

**Unit V:**

Homotopy Equivalence, Homotopy Groups of Spheres.[1] Chapter 6.

**Recommended Book:**

1. F. H. Croom : Basic Concepts of Algebraic Topology, Springer-Verlag New York.
2. W.S. Massey : Algebraic Topology, Springer -Verlag.
3. E.H. Spanier : Algebraic Topology : Mc Graw – Hill Book Company.

## TOPICS IN RING THEORY

**Pre-requisites:** Basic definitions and results concerning rings and fields.

### **Unit I:**

Rings and Ring Homomorphisms, Ideals Quotient Rings, Zero Divisors, Nilpotent Elements, Units.

### **Unit II:**

Prime Ideals and Maximal Ideals, Nilradical & Jacobson Radical Operations on Ideals, Extension & Contraction.

### **Unit III:**

Modules, Operation on Submodules, Direct Sum and Product of Modules, Restriction and Extension of Scalars.

### **Unit IV:**

Tensor product of modules, basic properties, Exactness Properties of Tensor Product, Algebras & Tensor Product of Algebras.

### **Unit-V:**

Rings and Modules of Fractions, Local Properties Extended and Contracted Ideals in Ring of Fractions. ( with Emphasis on Exercise) [1 chapter 1 to 3 ]

### **Books Recommended :**

1. Introduction to Commutative Algebra, Atiyah & I.G. Macdonald,  
Addison – Wesley Pub. Co

### **E-references:**

1. **Commutative Algebra Notes** Branden Stone  
[math.bard.edu/~bstone/commalg-notes/](http://math.bard.edu/~bstone/commalg-notes/)
2. **Commutative Algebra Lecture Notes** - Tata Institute of Fundamental  
[www.math.tifr.res.in/~anands/CA-Lecture%20notes.pdf](http://www.math.tifr.res.in/~anands/CA-Lecture%20notes.pdf)

## M 405

### ALGEBRAIC TOPOLOGY

**Pre-requisites: M113 & M213**

#### **UNIT I :**

Deformation retracts and homotopy type. Fundamental group of  $S^n$  for  $n > 1$ , and some surfaces. The Jordan separation theorem, the Jordan curve theorem, Imbedding graphs in plane.

[1] Chapter 9, sections 58 to 60 & Chapter 10, sections 61, 63 and 64.

#### **UNIT II :**

Free product of groups, Free groups, The Siefert- van Kampen theorem and its applications. Classification of surfaces : Fundamental groups of surfaces, Homology of surfaces, Cutting and pasting, Construction of Compact surfaces, The classification theorem.

[1] Chapter 11, sections 68 to 73 & Chapter 12, sections 74 to 78.

#### **Unit III :**

Equivalence of covering spaces, Covering transformations, The universal covering space, and its existence.: Homology groups of a simplicial complex : Simplicial complexes and simplicial maps, Homology groups, Homology groups of surfaces, Zero-dimensional homology, The homology of a cone, Relative homology, Homomorphisms induced by simplicial maps, Chain complexes and acyclic carriers. [1] Chapter 13. [2] Chapter 1, Sections 1 to 9, 12 & 13.

#### **UNIT IV :**

Relative homology : The exact homology sequence, Mayer-vietoris sequences, The Eilenberg-Steenrod axioms (without proofs). The singular homology groups, The axioms for Singular theory (without proofs), Mayer-Vietoris sequences, The isomorphism between simplicial and singular homology, CW complexes, The homology of CW complexes and application to Projective spaces and Lens spaces.

[2] Chapter 3, Sections 23 to 28 (relevant portions)

Chapter 4, Sections 29 to 34 & 37 to 40 (relevant portions)

**UNIT V :**

Cohomology : The Hom functor, Simplicial cohomology groups, Relative cohomology, The cohomology of free chain complexes, The cohomology of CW complexes, Cup products, Cohomology ring of surfaces.

[2] Chapter 5, Sections 41 to 49.

**Books recommended :**

[1] J.R. Munkres, Topology, Second edition, Prentice-Hall of India, 2000.

[2] J.R. Munkres, Elements of Algebraic topology, Addison-Wesley Publishing company, 1984.

**Analytic Number Theory**

**Unit I:** Characters of finite abelian groups, The character group, Dirichlet characters, Sums involving Dirichlet characters, Dirichlet's theorem on primes in arithmetic progressions. [1] Chapter 6, sections 6.5 to 6.10, Chapter 7

**Unit II:** Dirichlet series and Euler products, the function defined by Dirichlet series, The half-plane of convergence of a Dirichlet series, Integral formula for the coefficients of Dirichlet series, etc. [1] Chapter 11

**Unit III:** Properties of the gamma functions, Integral representations of Hurwitz zeta functions, Analytic continuation of Hurwitz zeta functions, Functional equation for the Riemann zeta function and properties of Riemann zeta functions etc. [1] Chapter 12

**Unit IV:** Analytic proof of prime number theorem. [1] Chapter 13

**Unit V:** Geometric representation of partitions, Generating functions of partitions, Euler's pentagonal number theorem, Euler's recursion formula for  $p(n)$ , Jacobi's triple product identity, The partition identity of Ramanujan. [1] Chapter 14

**Book Recommended:**

[1] T. M. Apostol, Introduction to Analytic Number Theory, Narosa Pub, House, 1989.

**Abstract Harmonic Analysis**

**Unit I** : Definition of a topological group and its basic properties. Subgroups and quotient groups. Product groups & projective limits. [1], Sections 4,5,6 of Chapter Two.

**Unit II** : Properties of topological groups involving connectedness. Invariant pseudo-metrics and separation axioms. Structure theory for compact and locally compact Abelian groups. Some special locally compact Abelian groups. [1], Sections 7,8,9,10 of Chapter Two.

**Unit III** : The Haar integral. Haar Measure. Invariant means defined for all bounded functions. Invariant means of almost periodic functions. [1], Chapter Four.

**Unit IV** : Convolutions, Convolutions of functions and measures. Elements of representation theory. Unitary representations of locally compact groups. [1], Chapter Five.

**Unit V** : The character group of a locally compact Abelian group and the duality theorem. [1], Sections 23,24 of Chapter Six.

**Recommended Book :**

1. Edwin Hewitt and Kenneth A. Ross, Abstract Harmonic Analysis-I, Springer-Verlag, Berlin, 1993.

**Reference :**

2. Lynn H. Loomis, An introduction to abstract harmonic analysis, D. Van Nostrand Co. Princeton.