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# A Hybrid Technique for Analysis of Low-Frequency Oscillation in Power System

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## ABSTRACT

Estimating the low-frequency oscillation in an interconnected power system is the most important requirement to keep the power system in a stable operating condition. This research work deals with a hybrid robust and accurate approach using a combination of Estimation of signal parameters via rotational invariant techniques (ESPRIT) and Prony algorithm to extract the low-frequency oscillatory modes present in the power system. The observation inspires the hybrid method that the true modes of the signal are present in any signal processing technique (for example, Prony algorithm) along with other fictitious modes regardless of the order of the power system. Moreover, this research obtained true modes by calculating Euclidean distance and applying the threshold value concept. The proposed technique is tested with different noise conditions and varying sampling rates of Phasor Measurement Unit (PMU) to check the proposed hybrid technique's robustness compared to Prony and the multiple ESPRIT method. Finally, the proposed method is applied to the real signal obtained from the Western Electricity Coordinating Council (WECC) network, and it estimates accurate and precise parameters compared to other methods. The accuracy for estimation of frequency and attenuation factor is calculated for the three-mode synthetic signal at a noise level of 10dB by the hybrid algorithm, multiple ESPRIT, and Prony algorithm, which shows that hybrid algorithm has minimum percentage error. Thus the proposed hybrid algorithm accurately estimates the parameters of low-frequency oscillation as compared to other existing methods without involving any fictitious modes.

*Keywords*: Attenuation factor, damping ratio, Prony algorithm, stability, synthetic signal

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## INTRODUCTION

Modern power systems are highly interconnected and often share useful information regarding the operation and control of the power system in a reliable manner. However, these interconnected power systems have a major challenge to maintain stability, such as small-signal stability, which contains low-frequency oscillations (Kundur, 1994). Small signal stability results in the system due to small disturbances due to a rise in rotor angle or rotor oscillation of continuously rising amplitude. Due to the non-linear behavior of the power system and during transient operation, it leads to ring-down oscillation, which needs to stabilize. Therefore, identifying critical modes is crucial, which helps design a controller that mitigates the poorly damped oscillations. In this direction, traditional techniques use Eigen values-based analysis using a linear time-invariant model in which a non-linear system is linearized at the operating point to identify low-frequency oscillatory modes of the power systems (Wang & Semlyen, 1990). Unfortunately, the oscillation properties change significantly due to variation in operating conditions of the system, which makes offline methods (Eigenvalue analysis) insufficient and meaningless for system operators. Recently, rapid development in the synchrophasor-based Wide Area Monitoring System (WAMS) and Phasor Measurement Units (PMU) facilitated the collection of the time-tagged data at the Phasor Data Concentrator (PDC), which enable the use of the online tool to estimate oscillatory modes present in the signal (Xie et al., 2005; Zhang et al., 2008).

Some common techniques which use the measurement-based estimation approach are Fast Fourier transform (FFT) (Girgis & Ham, 1980; Glickman et al., 2007), Kalman filter (Korba et al., 2003), Prony analysis (Amono et al., 1999; Hauer, 1991; Qi et al., 2007; Rai et al., 2016; Trentini et al., 2019; Trudnowski, 1994; Trudnowski et al., 1999; Wadduwage et al., 2015), ARMA (Wies et al., 2003), Continuous Wavelet Transform (CWT) (Kang & Ledwich, 1999; Rueda et al., 2011; Avdakovic et al., 2012), Hilbert-Huang transform (Laila et al., 2009), Matix-Pencil (Grant & Crow, 2011; Hua & Sarkar, 1990) and ESPRIT (Rai et al., 2014; Tripathy et al., 2011; Wang et al., 2014).

Among these methods, FFT is a fast method and has the robustness to noise which is easy to implement (Girgis & Ham, 1980; Glickman et al., 2007). However, this method does not work with low-resolution data and does not estimate the attenuation factor. Another method based on the recursion technique, the Kalman filter, has instability problems that make it difficult to use in the real application (Korba et al., 2003). Wavelet-based methods like CWT (Kang & Ledwich, 1999; Rueda et al., 2011) and Discrete Wavelet Transform (DWT) (Avdakovic et al., 2012) use multi-resolution analysis using the variable length of wavelet, which estimates modal information effectively on the stationary signal. The above techniques are easy to implement, but their accuracy depends on the shape of the mother wavelet and decomposition level. In HHT, Empirical mode decomposition (EMD) is used to make them work on non-stationary and non-linear data along with Hilbert spectral analysis (Laila et al., 2009). Unfortunately, estimation of frequency and damping ratio is accurate using this method if the modes obtained by EMD are mono-frequency components.

Moreover, EMD works only on the narrowband signal, which means the signal has adjacent frequency components or has a component that is not adjacent but has a large difference of energy intensity (Browne et al., 2008). Additionally, usage of EMD makes HHT based method slow and unable to fit in online mode estimation. On the other side, ARMA, which optimizes its parameter to estimate the model parameters more precisely, cannot estimate closely spaced modes (Wies et al., 2003). In Pierre et al. (1997), ambient noise is examined using Wiener-Hopf linear prediction. This method can estimate accurate details of frequencies but fails to predict precise information of damping. In literature, Zhou et al. (2008), regularized robust recursive least square (RLS) algorithm is used to estimate modal information. However, conceptually, it may diverge because regularized RLS have numerical instability problems.

Apart from these methods, Prony is a conventional method for modal analysis based on the frequency domain approach, estimating all required information like frequency, attenuation factor, phase, and amplitude (Qi et al., 2007). However, the usage of the Prony approach is limited because of its sensitivity to noise. Moreover, it cannot estimate true modes; the Prony algorithm obtains many fictitious modes. However, some efforts are made to improve the Prony method by separating signal and noise, but many fail due to improper model order estimation.

The ESPRIT method uses the shift-invariance property present in the signal to represent the auto-correlation matrix. Such a method is less sensitive to noise present in the signal but requires accurate information of modes to be extracted, otherwise leading to an incorrect finding of the true modes.

Normally, a simple order estimation technique is used with the ESPRIT algorithm to find true mode estimation. However, simple model order estimation often fails to accurately calculate the total number of modes in the power signal. To demonstrate the failure case of model order estimation, consider the following signal (Equation 1):

$$x_1 = e^{-0.0909t} cos(2\pi * 0.4t) + 0.9e^{-0.35t} cos(2\pi * 0.5t) + 0.7e^{-0.2001t} cos\left(2\pi * 0.6t + \left(\frac{\pi}{6}\right)\right) + 0.4e^{-0.666t} cos\left(2\pi * 1.1t + \left(\frac{\pi}{4}\right)\right) \quad [1]$$

The signal is sampled at 50Hz and corrupted with white Gaussian noise at 40dB. After applying Singular value decomposition (SVD) on the autocorrelation system, separating index K(i) can be calculated using Equation 2.

$$K(i) = \frac{\delta_1 + \delta_2 + \ldots + \delta_i}{\delta_1 + \delta_2 + \ldots + \delta_l} \quad [2]$$

where $l$ is the total number of singular values, and $\delta_i$ is the $i^{th}$ singular value of the autocorrelation matrix. The order is estimated by selecting index *i,* at which K(*i*) is closest to value 1. The plot of K(*i*) vs. index *i* is shown in Figure 1, which shows that the value of K(*i*) is closest to 1 at an index value of 5. However, the signal consists of only 4 frequency components. This incorrect result of order estimation fails ESPRIT to analyze the signal correctly.

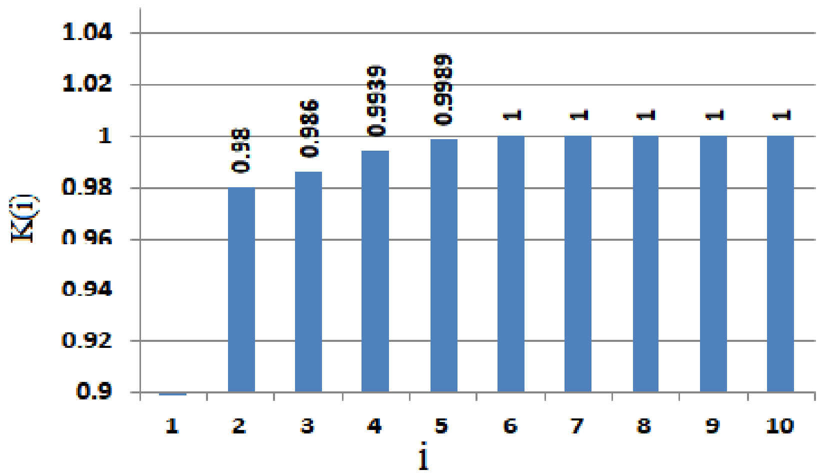


*Figure 1.* Plot of K(*i*) vs. *i* to estimate the order of the system

Several efforts have been attempted to improve the ESPRIT algorithm, but it lacks performance under high noise conditions. This research proposed a hybrid system based on ESPRIT and Prony algorithm to combine both methods' advantages, making it more robust to noise and estimating directly true modes from the signal without using any model order estimation technique. The proposed hybrid method is described in section 2, followed by simulation and the result analysis on various synthetic signals and real signals in section 3. At the end of section 4 conclusion of the research paper is discussed.

## PROPOSED HYBRID ALGORITHM

This section discussed the hybrid method using Prony and ESPRIT algorithms. As discussed, Prony and ESPRIT algorithm needs the information of exact (true) modes present in the signal to extract parameters of true modes. Moreover, the literature shows that improper mode estimation yields a failure case of the ESPRIT method. This research combined both methods and observed that true modes are present in both techniques with other fictitious modes to deal with the problem. Both the methods produced different fictitious modes, which can be removed easily by computing the Euclidean distance between them. It motivates us to combine two different methods. A detailed discussion of the Prony and ESPRIT methods can be obtained in the literature, respectively (Qi et al., 2007 & Tripathy et al., 2011). This section only shows the mathematical problem statement of mode estimation in the power system followed by only the main steps of both methods (i.e., Prony and ESPRIT). Later in this section, the true mode identification method using Euclidean distance will be shown.

In the power system to represent the signal, the linear combination of a damped sinusoid with white Gaussian noise is taken for analysis.

$$y(n) = s(n) + \omega(n) = \sum_{k=1}^{K} a_k e^{b_k n}(cos(n\omega_k + \varphi_k) + \omega(n)) \quad [3]$$

where s(n) and ω(n) are the signal component and zero-mean Gaussian noise, respectively. Moreover, amplitude, damping factor, damping frequency, initial phase, and the number of sinusoids are represented by $a_k$, $b_k$, $\omega_k$, $\varphi_{k,}$ and *k,* respectively. So the task of modal analysis is to find the best estimation of the parameters of Equation 3 so that the modes of oscillation in the power system are identified.

Equation 3 represents the time domain signal for the Prony algorithm (Hauer et al., 1990; Zhou et al., 2010). Theoretical derivations are well mentioned in the literature. As a summary, it follows the following steps.

1. The model that best fits the given signal is constructed by discrete linear prediction (LP).

2. Eigenvalues, roots of the characteristic polynomial are determined from the above prediction model.

3. Determine the least square solution of the equation to find the amplitude and phase angle of the modes.

It is noted that the Prony algorithm performance depends on the solution of *M* number of unknowns using (*N-M*) number of equations in above step-2, where *M* is the order of algorithm and *N* is the number of samples over the data windows, respectively. Hence, it is common to use the order *M* of the Prony should be taken as M ≤ (N/3) as given in Wadduwage et al. (2015) so that effect of noise is suppressed. So the order of ESPRIT and Prony algorithm in the proposed hybrid method is taken as M. However, the true modes are identified using Euclidean distance between the modes as given in next section **Hence, this research** of the paper irrespective of the order of the algorithm.

The main steps of the ESPRIT algorithm for estimating frequency can be summarised as given by Equations 4 to 11:

1. First construct Hankel matrix of order M from given signal to be analyzed which has a length of sample N. Mathematically, it is described as follows

$$X = \begin{bmatrix} X(0) & X(1)... & X\,(M-1) \\ X(1) & X(2)... & X(M) \\ ... & ... & ..... \\ X(N-M) & X(N-M+1) & ..X(N-1) \end{bmatrix} \quad [4]$$

2. From constructed Hankel matrix, derive autocorrelation matrix $R_x$ using given formula

$$R_x = \frac{1}{N-M}(X^H X) \quad [5]$$

3. Decompose the autocorrelation matrix using eigenvalue decomposition (via singular value decomposition) in the form of $R_x = UEV^*$ where E is a diagonal matrix that holds the Eigenvalues of $R_x$ in decreasing order.

4. Separate orthonormal eigenvectors from U based on the order of the model (M), which can be expressed as,

$$S = U(:,1:M) \quad [6]$$

5. Apply separation on matrix *S* to generate shifted submatrix as,

$$S_1 = [I_{M-1} 0]S \quad [7]$$

$$S_2 = [0 I_{M-1}]S \quad [8]$$

where $I_{M-1}$ is the identity matrix of order M-1.

6. These shifted submatrices are connected to matrix Φ by using shift-invariance condition expressed as $S_2 = S_1 \Phi$. Solve the below equation using least square estimation to obtain matrix Φ.

$$\Phi = \left(S_1{}^H S_1\right)^{-1}\left(S_1{}^H S_2\right) \quad [9]$$

7. The frequency and damping factor of a signal can be estimated from eigenvalue $\lambda_i$ of the matrix Φ as,

$$f_i = f_s * \left(\frac{imag(\log \lambda_i)}{2\pi}\right) \quad [10]$$

$$\delta_i = f_s * real(\log \lambda_i) \quad [11]$$

Consider a synthetically generated signal **Y** with three different modes, as shown in Table 1 with the sampling frequency of 60 Hz. Next, consider two different length signals from windows at 1–20 sec and 2–18 sec, respectively. Next, apply the ESPRIT algorithm on 20-sec window length and the Prony algorithm on 16–sec window length. In this case, the model order of ten for both algorithms, which estimate five modes with the positive frequency, is shown in Table 2.

Table 1

*Details of three-mode synthetic signal*

| Mode | Frequency | Attenuation Factor | Amplitude | Damping Ratio % |
|---|---|---|---|---|
| 1 | 0.25 | -0.1102 | 1 | 7.0169 |
| 2 | 0.39 | -0.1596 | 1 | 6.5143 |
| 3 | 0.7 | -0.2199 | 0.5 | 5.0007 |

Table 2

*Mode Estimation Result of Synthetic Signal from Table1 using ESPRIT and Prony method*

| | ESPRIT | | | Prony | | |
|---|---|---|---|---|---|---|
| Mode | Frequency (Hz) | Attenuation Factor | Damping Ratio (%) | Frequency (Hz) | Attenuation Factor | Damping Ratio (%) |
| 1 | 0.7000 | -0.2199 | 5.0007 | 0.7003 | -0.2198 | 4.9950 |
| 2 | 0.2500 | -0.1102 | 7.0169 | 0.2500 | -0.1102 | 7.0179 |
| 3 | 0.3900 | -0.1596 | 6.5143 | 0.3902 | -0.1596 | 6.5103 |
| 4 | 30.0152 | -21.0835 | 11.1816 | 25.4421 | -28.1292 | 17.5997 |
| 5 | 11.1988 | -14.2470 | 20.2514 | 17.9317 | -26.9529 | 23.9269 |

The core logic behind the proposed hybrid method is the observation that true modes appear consistently in both methods, whatever be the model order. However, due to the dynamic behavior of power systems and noise, other fictitious modes are also inherently present, not estimated the same in different methods. So it can easily be removed by applying a threshold on the closest distance with modes estimated by other methods.

This paper used Euclidean distance to find the distance between each mode estimated from ESPRIT and Prony methods. Assume that $p_1$ and $p_2$ modes are identified using ESPRIT and Prony methods. The modes can be identified as fictitious and rejected if the condition given in Equation 12 does not satisfy them. If,

$$\sqrt{(f_i - f_j)^2 + (\sigma_i - \sigma_j)^2} \le \tau \qquad i = 1,2,3,\dots p_1 \;\&\; j = 1,2,3,\dots p_2 \qquad [12]$$

Here τ is the threshold value to pick out true modes, taken as 0.02. Finally, the average estimated value is taken to reduce calculation error in the solution using Equations 13 and 14. Here σ indicates the real part of the eigenvalue.

$$f = \frac{f_1 + f_2}{2} \qquad [13]$$

$$\sigma = \frac{\sigma_1 + \sigma_2}{2} \qquad [14]$$

The block diagram of the proposed algorithm to estimate the exact true modes in the signal is represented as in Figure 2. It has to be noted that complex eigenvalues occur in conjugate pairs, and each pair denotes the single mode of oscillation. Furthermore, this paper is concerned with low-frequency oscillation analysis, which means the higher frequencies from the solution can be omitted. Hence, it is only required to check Equation 12 for positive and frequencies lower than 5 Hz.

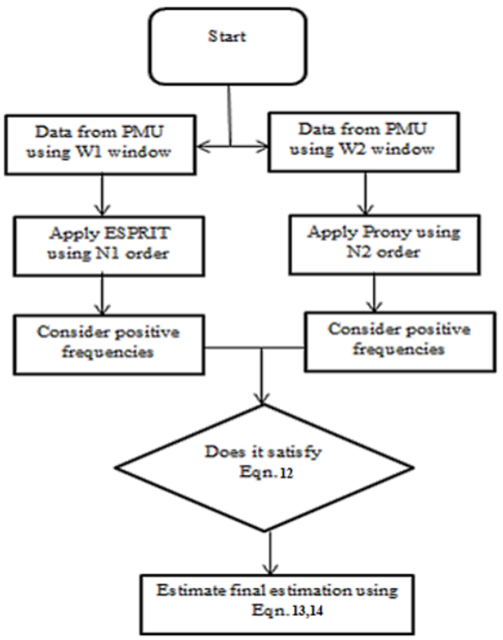


*Figure 2.* Block diagram representation of the proposed algorithm

## RESULTS AND DISCUSSION

This research analyzes the hybrid method's simulation result and other current state-of-the-art methods for different scenarios. The complete simulation has been done on MATLAB software under the Windows environment.

In the first case, this research analyzed synthetic signals with the same damping ratio and large separation in frequency components. The details of the synthetically generated signal are given in Table 3. The sampling frequency is 60 Hz in this case.

Here, to check the performance of the proposed algorithm under different noise levels, additive white Gaussian noise (AWGN) is added, and the robustness of the different methods is checked. As mentioned previously, both Prony and ESPRIT methods are used to identify the hybrid approach's true modes. Moreover, the performance of multiple ESPRIT and Prony methods is presented in Table 6, which justifies that the hybrid algorithm is more accurate as compared to Prony and ESPRIT methods under high noise conditions when there is a large separation in frequency components and closed damping ratio.

Table 3

*Parameters of the synthetic signal with the same damping ratio*

| Mode No. | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio % |
|---|---|---|---|---|
| 1 | 0.2 | -0.1 | 1 | 7.9592 |
| 2 | 0.8 | -0.4 | 1 | 7.9592 |

Table 6 shows that the hybrid method is more robust than the Prony and multi ESPRIT methods in the varying condition of noise. In high noise conditions (i.e., 10dB noise level) hybrid method predicts accurate results compared to other methods. It is noted that the Prony method estimates all modes, including fictitious modes, which is further needed to be removed. However, this result only considers true mode estimation from the Prony method ignoring fictitious modes obtained from the analysis.

Apart from tabular measurement, visual estimated signals and the original noisy signal are analyzed in Figure 3. It is clear from the analysis that the hybrid method under the high level of noise condition can also estimate the exact information regarding the low-frequency oscillation parameters.

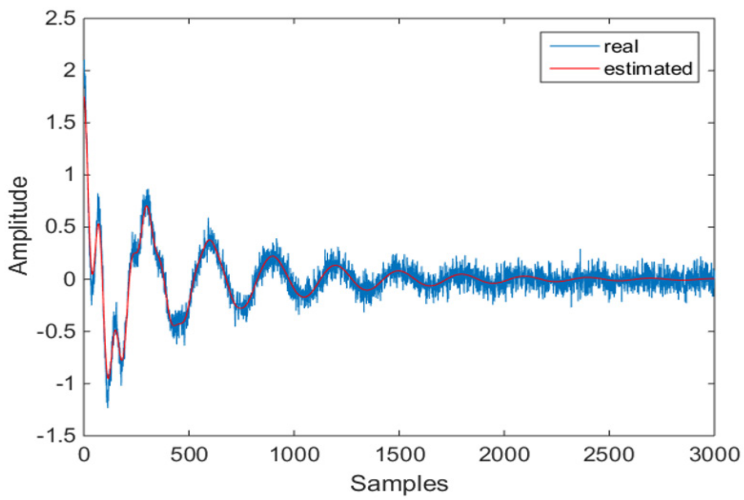


*Figure 3.* Estimated signal using the proposed hybrid method given in Table 3 with noise level 10dB

In the second case, this research generates a signal having two modes with closed frequency components and a twice damping ratio. Table 4 shows the detail of the synthetically generated signal. In this case, the sampling frequency is 60 Hz, and the estimated values are given in Table 7.

Table 4

*Parameters of the synthetic signal with closed frequency components*

| Mode No. | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio % |
|---|---|---|---|---|
| 1 | 0.5 | -0.25 | 1 | 7.9592 |
| 2 | 0.6 | -0.15 | 1 | 3.9796 |

From Table 7 also, it is observed that the proposed hybrid method shows robust performance under varying noise conditions and is even capable of estimating the parameters of the low-frequency oscillation signal, which consists of almost the same frequency components. For example, Figure 4 shows how closely the proposed algorithm can estimate the true signal from the synthetic signal having a noise level of 10dB using the proposed hybrid algorithm.

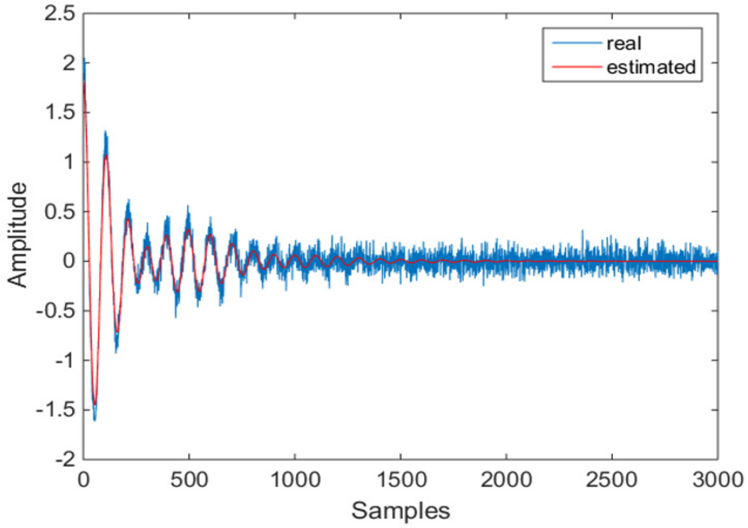


*Figure 4.* Estimated signal using the proposed hybrid method given in Table 4 with noise level 10dB

Preceding all results are taken under 60 Hz of sampling frequency. In practice, the reporting rate of PMU varies from 10 Hz to 100 Hz. Literature shows that some techniques have limitations to work under a certain sampling frequency range. Fortunately, the proposed method can work under even varying sampling rates also. In order to check its performance under such conditions, this research conducted the experiment in which it considered the same synthetic signal from Table 4 and tried to estimate true modes using the proposed hybrid method on different sampling frequencies ranging from 30 Hz to 60 Hz. Table 5 shows the result of the hybrid method under different sampling frequencies (PMU reporting rate) under 30 dB noise level. The proposed hybrid method provides accurate results for a range of PMU reporting rates, facilitating its application with any system with different PMU reporting rates.

In the next case, this research considers synthetic signal with parameters is shown in Table 1 with 60 Hz sampling frequency. This synthetic signal has three modes, with three frequency components and a damping ratio. In order to check the robustness of the proposed hybrid method, the signal is corrupted with distinct levels of additive white Gaussian noise (AWGN) varying from 50 dB to 10 dB. The result analysis is shown in Table 8, which

Table 5

*Mode estimation by proposed hybrid method with varying PMU reporting rate*

| PMU Reporting Rate | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio% |
|---|---|---|---|---|
| 60Hz | 0.5 | -0.249 | 0.99 | 7.9274 |
| | 0.6 | -0.149 | 1 | 3.9531 |
| 40 Hz | 0.49 | -0.247 | 0.99 | 8.0242 |
| | 0.59 | -0.147 | 0.99 | 3.9661 |
| 30 Hz | 0.48 | -0.245 | 1.01 | 8.1251 |
| | 0.58 | -0.145 | 0.99 | 3.9796 |

indicates how the proposed hybrid method accurately estimates low-frequency oscillatory modes of the synthetic signal having three frequency components even in the presence of noise and has better performance as compared to multiple ESPRIT and Porny algorithms.

The previous section shows the performance of the proposed hybrid method, multiple ESPRIT, and Porny algorithm in Tables 6, 7, and 8, respectively, for a different set of synthetic signals with varying noise levels. However, those results are one-time measurements that may vary every time due to the randomness of noise. Therefore, it is required to estimate accurate modes all the time despite the randomness of noise. It means estimation should have low variation in the output result all the time. This research performs a Monte Carlo simulation with 1000 samples and 300 samples for the proposed hybrid and ESPRIT methods, respectively, to test the performance of the proposed method. Figure 5 shows the histogram of estimated frequency versus the number of counts for multiple ESPRIT methods. From the histogram, it is observed that the multiple ESPRIT method estimate values with high variance. Hence, the research based on multiple ESPRIT needs to estimate several times and average it to estimate final results. Unfortunately, an improper control action may occur if the true modes are not identified in the real scenario. Figure 6 shows the histogram of estimated frequency versus the number of counts for the proposed hybrid method, which shows the very little variance in the result, making it suitable for real-time application.

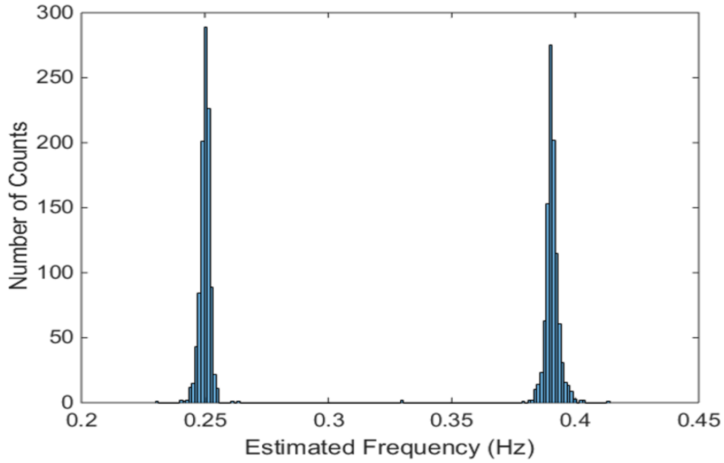


*Figure 5.* Histogram of estimated frequency by multiple ESPRIT method

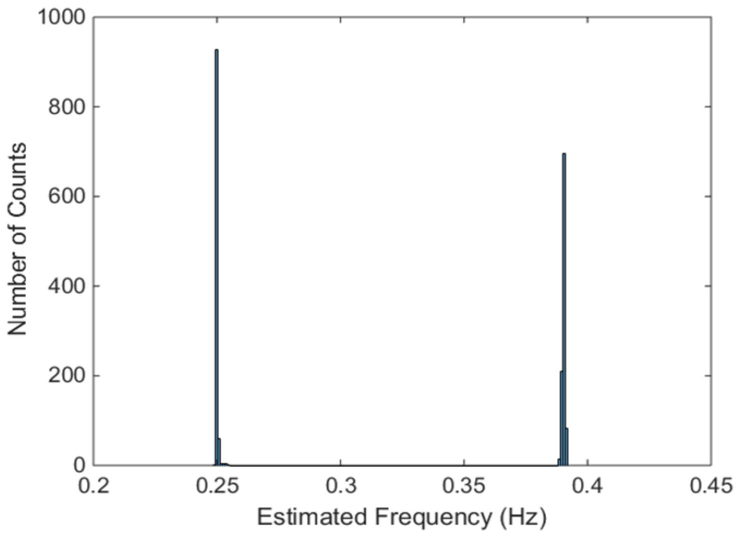


*Figure 6.* Histogram of estimated frequency by the proposed hybrid method

Table 6

*Mode Estimation on Synthetic Signal having same damping ratio*

| Method | | No noise | | 50dB | | 30dB | | 20dB | | 10dB | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Hybrid | Freq (Hz) | 0.200 | 0.800 | 0.199 | 0.800 | 0.199 | 0.799 | 0.197 | 0.796 | 0.195 | 0.797 |
| | Attenuation Factor | -0.100 | -0.399 | -0.099 | -0.400 | -0.099 | -0.399 | -0.097 | -0.398 | -0.097 | -0.396 |
| | Amplitude | 1.000 | 1.000 | 1.000 | 1.000 | 0.990 | 0.990 | 0.970 | 0.960 | 1.030 | 0.960 |
| | Damping Ratio % | 7.9592 | 7.9394 | 7.9193 | 7.9592 | 7.9193 | 7.9493 | 7.8380 | 7.9592 | 7.9184 | 7.9093 |
| Multiple ESPRIT | Freq (Hz) | 0.199 | 0.800 | 0.199 | 0.799 | 0.197 | 0.798 | 0.179 | 0.781 | 0.167 | 0.685 |
| | Attenuation Factor | -0.100 | -0.395 | -0.098 | -0.398 | -0.098 | -0.397 | -0.08 | -0.369 | -0.071 | -0.349 |
| | Amplitude | 0.990 | 1.001 | 0.980 | 1.001 | 1.010 | 0.990 | 1.030 | 0.960 | 1.030 | 0.850 |
| | Damping Ratio % | 7.9992 | 7.8598 | 7.8393 | 7.9294 | 7.9188 | 7.9194 | 7.1144 | 7.5210 | 6.7677 | 8.1103 |
| Prony | Freq (Hz) | 0.199 | 0.799 | 0.198 | 0.797 | 0.196 | 0.797 | 0.169 | 0.688 | 0.159 | 0.585 |
| | Attenuation Factor | -0.099 | -0.398 | -0.098 | -0.398 | -0.097 | -0.396 | -0.075 | -0.319 | -0.057 | -0.250 |
| | Amplitude | 1.001 | 1.001 | 0.990 | 0.990 | 1.020 | 0.980 | 1.050 | 0.910 | 1.050 | 0.750 |
| | Damping Ratio % | 7.9193 | 7.9294 | 7.8789 | 7.9493 | 7.8780 | 7.9093 | 7.0644 | 7.3808 | 5.7066 | 6.8028 |

Table 7

*Mode Estimation on Synthetic Signal having closed frequency components*

| Method | | No noise | | 50dB | | 30dB | | 20dB | | 10dB | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Hybrid | Freq (Hz) | 0.499 | 0.600 | 0.500 | 0.599 | 0.499 | 0.599 | 0.497 | 0.596 | 0.495 | 0.594 |
| | Attenuation Factor | -0.249 | -0.150 | -0.250 | -0.150 | -0.250 | -0.149 | -0.248 | -0.147 | -0.246 | -0.145 |
| | Amplitude | 0.990 | 1.000 | 0.990 | 0.990 | 1.010 | 0.990 | 1.010 | 0.990 | 1.020 | 0.970 |
| | Damping Ratio % | 7.9433 | 3.9796 | 7.9592 | 3.9863 | 7.9752 | 3.9597 | 7.9432 | 3.9262 | 7.9110 | 3.8858 |
| Multiple ESPRIT | Freq (Hz) | 0.500 | 0.599 | 0.499 | 0.599 | 0.495 | 0.593 | 0.471 | 0.587 | 0.455 | 0.577 |
| | Attenuation Factor | -0.249 | -0.149 | -0.251 | -0.149 | -0.248 | -0.149 | -0.229 | -0.139 | -0.221 | -0.135 |
| | Amplitude | 1.010 | 1.010 | 0.990 | 0.990 | 0.990 | 1.010 | 1.020 | 0.970 | 1.040 | 0.910 |
| | Damping Ratio % | 7.9274 | 3.9597 | 8.0071 | 3.9597 | 7.9753 | 3.9998 | 7.7396 | 3.7695 | 7.7318 | 3.7244 |
| Prony | Freq (Hz) | 0.499 | 0.599 | 0.498 | 0.599 | 0.491 | 0.592 | 0.451 | 0.567 | 0.431 | 0.359 |
| | Attenuation Factor | -0.249 | -0.15 | -0.248 | -0.148 | -0.246 | -0.145 | -0.211 | -0.121 | -0.157 | -0.115 |
| | Amplitude | 0.990 | 0.990 | 0.990 | 0.990 | 0.980 | 1.020 | 1.020 | 0.970 | 1.090 | 0.850 |
| | Damping Ratio % | 7.9433 | 3.9863 | 7.9273 | 3.9331 | 7.9755 | 3.8990 | 7.4475 | 3.3971 | 5.7986 | 5.0992 |

Table 8

*Mode Estimation on Synthetic Signal having three modes*

| Method | Hybrid | | | | Multiple ESPRIT | | | | Prony | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Noise | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio% | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio% | Freq (Hz) | Attenuation Factor | Amplitude | Damping Ratio% |
| No noise | 0.7000 | -0.2199 | 0.4990 | 5.0007 | 0.7002 | -0.2198 | 0.4990 | 4.9978 | 0.7003 | -0.2198 | 0.4980 | 4.9950 |
| | 0.3900 | -0.1596 | 1.0000 | 6.5143 | 0.3901 | -0.1596 | 1.0000 | 6.5118 | 0.3902 | -0.1596 | 0.9970 | 6.5103 |
| | 0.2500 | -0.1102 | 0.9970 | 7.0169 | 0.2500 | -0.1101 | 0.9970 | 7.0108 | 0.2500 | -0.1100 | 1.0001 | 7.0052 |
| 50dB | 0.7000 | -0.2170 | 0.4990 | 4.9347 | 0.7000 | -0.2150 | 0.5000 | 4.8893 | 0.7001 | -0.2195 | 0.4970 | 4.9896 |
| | 0.3900 | -0.1601 | 1.0000 | 6.5343 | 0.3890 | -0.1601 | 1.0000 | 6.5511 | 0.3900 | -0.1594 | 0.9975 | 6.5054 |
| | 0.2500 | -0.1098 | 0.9970 | 6.9912 | 0.2500 | -0.1098 | 0.9980 | 6.9912 | 0.2500 | -0.1100 | 1.0120 | 7.0052 |
| 30dB | 0.6974 | -0.2194 | 0.4975 | 5.0068 | 0.6898 | -0.2224 | 0.4970 | 5.1331 | 0.6998 | -0.2324 | 0.4980 | 5.2873 |
| | 0.3954 | -0.1588 | 1.0173 | 6.3951 | 0.3900 | -0.1594 | 0.9989 | 6.5054 | 0.3984 | -0.1618 | 1.0083 | 6.4668 |
| | 0.2500 | -0.1100 | 1.0012 | 7.0052 | 0.2514 | -0.1141 | 1.0162 | 7.2212 | 0.2504 | -0.1132 | 1.0262 | 7.1928 |
| 20dB | 0.7011 | -0.2252 | 0.4960 | 5.1130 | 0.6961 | -0.2142 | 0.4950 | 4.8982 | 0.6971 | -0.2092 | 0.4970 | 4.7770 |
| | 0.3904 | -0.1614 | 1.0137 | 6.5818 | 0.3874 | -0.1624 | 1.1107 | 6.6739 | 0.3874 | -0.1624 | 1.1105 | 6.6739 |
| | 0.2505 | -0.1147 | 1.0216 | 7.2863 | 0.2495 | -0.1067 | 0.9716 | 6.8051 | 0.2345 | -0.1056 | 0.9516 | 7.1657 |
| 10dB | 0.6980 | -0.2238 | 0.4910 | 5.1048 | 0.6880 | -0.2038 | 0.4890 | 4.7162 | 0.7058 | -0.2015 | 0.4890 | 4.5456 |
| | 0.3896 | -0.1582 | 0.9083 | 6.4623 | 0.4011 | -0.1703 | 1.1057 | 6.7573 | 0.4011 | -0.1733 | 1.1057 | 6.8763 |
| | 0.2495 | -0.1134 | 1.0380 | 7.2368 | 0.2525 | -0.1048 | 0.9576 | 6.6032 | 0.4011 | -0.1668 | 0.9476 | 6.6184 |

The accuracy of the proposed hybrid algorithm is checked for the synthetic signal having three modes at 10dB noise level. From the analysis given in Table 8 of the synthetic signal having three modes with frequency and attenuation factor as 0.25Hz and -0.1102, 0.39 Hz and -0.1596, 0.7 Hz and -0.2199, respectively, the percentage error for estimating frequency component of 0.39 Hz at 10dB noise level by hybrid algorithm, multiple ESPRIT, and Prony algorithm is 0.1%, 2.8%, and 2.8% whereas the percentage error for estimating the attenuation factor of -0.1596 at 10dB noise level by hybrid algorithm, multiple ESPRIT, and Prony algorithm is 0.87%, 6.7% and 8.5% which indicates that the proposed algorithm is having higher accuracy as compared to other methods even at 10dB noise level for three-mode signal.

In the subsequent section, this research tested the proposed hybrid method on the WECC system from probe test data obtained on 14th September 2005. Since this research is interested in transient analysis, it considers two windows, as shown in Figure 7. Analysis windows correspond to data obtained after two sequential probing of ±125 MW, as shown in Figure7. Both analysis window is also corrupted by adding 30 dB AWGN white noise.

The estimation of frequency and damping ratio (ζ) is carried out on these two windows using the proposed hybrid method along with HTLS (Philip & Jain, 2018b), EMO-ESPRIT (Philip & Jain, 2018a), and TLS-ESPRIT (Tripathy et al., 2011) methods to compare the performance. The estimation result is shown in Table 9, including the estimated value as suggested by Philip and Jain (2018a) and Rai et al. (2014). Therefore, it can be inferred that the hybrid model accurately estimates frequency and damping ratio values on WECC probe data which is very close as given in Philip & Jain (2018a) and Rai et al. (2014). In the next part, parameters estimation on WECC data is obtained under the varying noise level ranging from 50 dB to 10 dB, and the result is tabulated in Table 10. The developed hybrid method is robust and reliable with various noise conditions, as discussed and tested earlier. The observation can be inferred from Table 10, where the hybrid method estimates damping ratio and frequency accurately from two different windows despite the presence of noise.

Table 9

*Mode estimation of frequency and damping ratio of WECC Probe data*

| Window | Suggested value is given in Philip and Jain (2018a) and Rai et al. (2014) | | Proposed Hybrid Method | | HTLS (Philip & Jain, 2018b) | | TLS-ESPRIT (Tripathy et al., 2011) | | EMO-ESPRIT (Philip & Jain, 2018a) | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Freq (Hz) | ζ | Freq (Hz) | ζ | Freq (Hz) | ζ | Freq (Hz) | ζ | Freq (Hz) | ζ |
| Window-1 | 0.318 | 8.30 | 0.3186 | 8.33 | 0.3183 | 8.39 | 0.3259 | 6.86 | 0.3207 | 8.30 |
| Window-2 | 0.318 | 8.30 | 0.3141 | 8.11 | 0.316 | 8.11 | 0.3151 | 7.78 | 0.3149 | 7.88 |

Table 10

*Estimation of damping ratio and frequency of WECC probe data under various noise levels by a hybrid method*

| Window | 50 dB | | 35 dB | | 10 dB | |
|---|---|---|---|---|---|---|
| | Freq (Hz) | ζ | Freq (Hz) | ζ | Freq (Hz) | ζ |
| Window-1 | 0.3186 | 8.31 | 0.3176 | 8.23 | 0.3157 | 8.23 |
| Window-2 | 0.3171 | 8.21 | 0.3161 | 8.11 | 0.3138 | 8.09 |

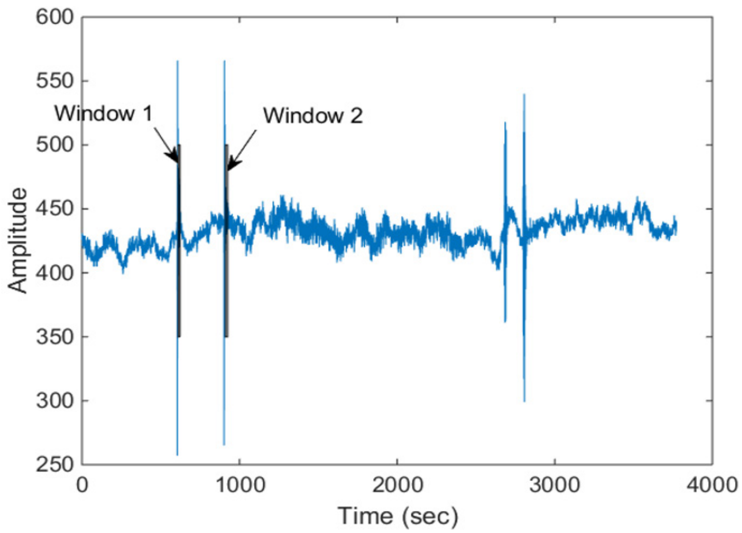


*Figure 7.* Test probe data of WECC system on 14th September 2005 with consideration of two windows to be analyzed.

## CONCLUSION

In order to estimate true modes in the power system due to low-frequency oscillation, exact model order estimation is required for signal processing techniques like Prony or ESPRIT, which has been overcome in this research paper. This research work proposed a hybrid method based on the Prony and ESPRIT algorithm to find true modes from the low-frequency oscillatory signal based on the Euclidean distance concept irrespective of the model order of the algorithm. The proposed method is motivated by observing that true modes are present in both methods and other fictitious modes. However, it is encouraged to use different methods to remove fictitious modes accurately. This research has shown the robustness of the hybrid method with different noise levels and varying reporting rates of PMU. It is also shown that the discussed method is more reliable compared to multiple ESPRIT methods and estimate accurate modes in single and multiple trials. Moreover, the proposed method on WECC probe data was tested with the various noise levels and the current state-of-the-art methods, which show the outstanding performance of the proposed method. Hence, the proposed method is more robust and reliable to use in practice under any conditions in terms of varying PMU sampling rates and different noise levels.

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